

# Tenacious domain walls in supersymmetric QCD

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(Received 19 June 2001; published 27 November 2001)

We study the structure of the tenacious (existing for all values of masses of the matter fields) Bogomol'nyi-Prasad-Sommerfield domain walls interpolating between different chirally asymmetric vacua in supersymmetric QCD in the limit of large masses. We show that the wall consists in this case of three layers: two outer layers form a “coat” with the characteristic size  $\sim \Lambda_{\text{SYM}}^{-1}$  and there is also the core with width  $\sim m^{-1}$ . The core always carries a significant fraction of the total wall energy. This fraction depends on  $N_f$  and on the “windings” of the matter fields.

DOI: 10.1103/PhysRevD.64.125008

PACS number(s): 11.27.+d, 12.60.Jv

## I. INTRODUCTION

The dynamics of supersymmetric gauge theories with or without additional matter multiplets has attracted the attention of theorists since the beginning of the 1980's. It is very well known [1] that the pure supersymmetric Yang-Mills (SYM) theory, as well as a class of theories involving extra matter supermultiplets (SQCD), based on the  $SU(N_c)$  gauge group, involve  $N_c$  different chirally asymmetric vacuum states characterized by the different phases of the gluino condensate

$$\langle \text{Tr } \lambda^2 \rangle = \sum e^{2\pi i k / N_c}, \quad k = 0, \dots, N_c - 1. \quad (1)$$

It was argued recently [2] that on top of  $N_c$  chirally asymmetric vacua (1), a chirally symmetric vacuum with a zero value of the condensate also exists.

The presence of different degenerate physical vacua in the theory implies the existence of domain walls—static field configurations depending only on one spatial coordinate ( $z$ ) which interpolate between one of the vacua at  $z = -\infty$  and another one at  $z = \infty$  and minimizing the energy functional. As was shown in [3,4], in many cases the energy density of these walls can be found exactly due to the fact that the walls present the Bogomol'nyi-Prasad-Sommerfield- (BPS-) saturated states:

$$\epsilon = \frac{N_c}{8\pi^2} |\langle \text{Tr } \lambda^2 \rangle_\infty - \langle \text{Tr } \lambda^2 \rangle_{-\infty}|, \quad (2)$$

where the subscript  $\pm\infty$  marks the values of the gluino condensate at spatial infinities. The right side of Eq. (2) presents an absolute lower bound for the energy of any field configuration interpolating between different vacua.

The relation (2) is valid *assuming* that the wall is BPS saturated. However, whether such a BPS-saturated domain wall exists or not is a nontrivial dynamic question which can be answered only in a specific study of a particular theory of interest.

In Refs. [5,6] this question was studied in theories involving  $N_f = N_c - 1$  different quark and squark flavors. (Each flavor corresponds to a pair of chiral supermultiplets  $S_f$  and  $S'_f$  with opposite chiralities.) These theories are distinguished by the fact that the vacuum expectation values of squark fields

give the mass to *all* gauge bosons of the group  $SU(N_c)$  through the Higgs mechanism. Also, when the mass  $m$  of the matter fields is small  $m \ll \Lambda_{\text{SQCD}}$ , the effective coupling constant is small and the dynamics of the theory can be analyzed perturbatively.

In particular, the low energy dynamics of the theory in the Higgs phase is described by the Affleck-Dine-Seiberg effective Lagrangian for the composite chiral superfields  $\mathcal{M}_{ij} = 2S'_i S_j$  [1]. It has the Wess-Zumino nature with the superpotential

$$W = -\frac{2(N_c - N_f)}{3(\det \mathcal{M})^{1/(N_c - N_f)}} - \frac{m}{2} \text{Tr } \mathcal{M}. \quad (3)$$

When writing Eq. (3), we assumed that all quark/squark flavors are endowed with the same small mass  $m$ . For future purposes, we have left  $N_f$  as a free parameter (with the restriction  $N_f < N_c$ ). From now on we set  $\Lambda_{\text{SQCD}} = 1$ .

It is not difficult to see that the corresponding potential for the scalar components  $\mu_{ij}$  of  $\mathcal{M}_{ij}$ ,

$$U(\mu_{ij}, \bar{\mu}_{ij}) = \sum_{ij} \left| \frac{\partial W}{\partial \mu_{ij}} \right|^2, \quad (4)$$

has  $N_c$  degenerate supersymmetric minima. The chiral condensate at the vacua is given by the relation

$$\langle \text{Tr } \lambda^2 \rangle_{\text{vac}} = \frac{16\pi^2}{N_c} W(\text{vac}). \quad (5)$$

It has the form (1) with

$$\Sigma = \frac{32\pi^2}{3} \left( \frac{3m}{4N_f} \right)^{N_f/N_c}. \quad (6)$$

The ADS effective Lagrangian has a Wilsonian nature in the sense that the characteristic mass of the Higgs field excitations it describes is of order  $m$ , which is much smaller than the mass of the heavy gauge bosons. It is important to understand, however, that it was derived under the *assumption* that the relevant values of  $|\mu_{ij}|$  are large. This is not true near the chirally symmetric vacuum, where  $\langle \mu_{ij} \rangle = 0$ , and

there is no wonder that the latter is not seen in the ADS effective Lagrangian framework (see [4] for a detailed discussion).

Adopting the simplest ansatz

$$\mathcal{M}_{ij} = \delta_{ij} X^2 \quad \text{and hence} \quad \mu_{ij} = \delta_{ij} \chi^2 \quad (7)$$

and adding to the potential (4) the kinetic term  $|\partial_\mu \chi|^2$ , we can obtain the BPS wall solutions which interpolate between different chirally asymmetric vacua. For the theory with  $N_c = 2$ ,  $N_f = 1$ , an analytic solution exists [4]. In other cases the solutions can be found numerically (see Ref. [6] and Sec. III of this paper).

When the mass  $m$  is not small, the lightest states in the spectrum have glueball or glueballino nature and one cannot write down a truly Wilsonian effective Lagrangian. However, the situation is better here than, say, in pure nonsupersymmetric Yang-Mills theory. In our case, the *potential* part of the effective Lagrangian is rigidly fixed by symmetry considerations. It is expressed in terms of  $\mathcal{M}_{ij}$  and of the colorless chiral superfield

$$S \equiv \Phi^3 = \frac{3}{32\pi^2} \text{Tr}\{W_\alpha W^\alpha\} \quad (8)$$

representing the gauge sector. The lowest component of  $S$  is proportional to  $\text{Tr} \lambda^2$ . Supersymmetry and the exact relations for the conformal and chiral anomalies dictate the following form of the superpotential [7]:

$$W = \frac{2}{3} \Phi^3 [\ln(\Phi^{3(N_c - N_f)} \det \mathcal{M}) - (N_c - N_f)] - \frac{m}{2} \text{Tr} \mathcal{M}. \quad (9)$$

However, the kinetic term of the Lagrangian is not fixed rigidly, even though the requirement of the absence of an extra dimensionful parameter imposes significant restrictions. The simplest choice is

$$\mathcal{L}^{\text{kin}} = \int d^4\theta [\bar{\Phi} \Phi + \mathcal{K}(\bar{\mathcal{M}}, \mathcal{M})], \quad (10)$$

where the Kähler potential  $\mathcal{K}(\bar{\mathcal{M}}, \mathcal{M})$  is the same as in the ADS Lagrangian. It is obtained from the term  $\sum_i (\bar{S}_i S_i + \bar{S}'_i S'_i)$  in the original SQCD Lagrangian, which describes physics adequately for large values of moduli. The sum of Eq. (10) and  $\text{Re}[\int d^2\theta W(\Phi, \mathcal{M}_{ij})]$  is called the *Taylor-Veneziano-Yankielowicz* (TVY) effective Lagrangian.

Domain walls in supersymmetric QCD with  $N_f = N_c - 1$  were studied in the TVY framework in Refs. [4–6]. The results are the following.

(1) On top of the chirally asymmetric vacua (1), the system also enjoys the chirally symmetric vacuum with  $\langle \phi^3 \rangle = \langle \mu_{ij} \rangle = 0$ .

(2) For any value of mass there are “real” (i.e., without essential complex dynamics) Bogomol’nyi-Prasad-Sommerfield (BPS) solutions interpolating between the chirally symmetric vacuum and each chirally asymmetric one.

(3) For small masses there are also *two different* complex BPS wall solutions interpolating between adjacent chirally asymmetric vacua. In the limit  $m \rightarrow 0$ , one of these solutions (the “upper BPS branch”) goes over to the BPS solution in the ADS effective Lagrangian. Another solution (the lower BPS branch) passes near the chirally symmetric minimum and is not described by the ADS Lagrangian.

(4) When the mass grows, the two BPS branches approach each other. They fuse at some critical value  $m_*$ . For  $m > m_*$ , there is no BPS solution at all. A domain wall still exists in the range  $m_* < m < m_{**}$ , but it is no longer BPS saturated. At  $m > m_{**}$ , there are no such walls whatsoever. We have studied the theories with  $N_c = 2, 3, 4$ . The analysis was later extended to larger  $N_c$  (up to  $N_c = 8$ ) [8]. The critical value  $m_*$  falls off rapidly with increasing  $N_c$ , while  $m_{**}$  is roughly constant.

Recently [9], theories with an arbitrary number of flavors were analyzed along the same lines. It was found that, at  $N_f < N_c/2$ , we have a completely different picture; namely, there is only one complex BPS branch and it exists for *any* value of mass. This finding was confirmed in [8]. A qualitative explanation of this phenomenon was given in [9]. In particular, the limit  $m \rightarrow \infty$  was explored. It was noted that the profile of the wall acquires for large masses a universal form, which can be found in the framework of the VY effective Lagrangian for pure supersymmetric Yang-Mills theory.

In addition, it was noticed that such “tenacious” domain walls also exist in the theories with  $N_f = N_c - 1$ , if one relaxes the requirement (7). Assuming that  $\mathcal{M}_{ij}$  is still diagonal, but its different components are not equal, one is able to construct complex domain walls that persist for arbitrary large masses.

In this paper we make two remarks. We note that, when  $m$  is large, “tenacious” walls have a complex “matryoshka” structure. It involves the VY “coat,” for which heavy matter fields decouple, and the core, where the moduli  $\mu_{ij}$  are “alive.” The core has small width  $\sim 1/m$ , but, as the fields  $\mu_{ij}$  change rapidly there and the energy density is big, it carries a significant fraction of the total wall energy, which we calculate.

Another remark is that the flavor asymmetric walls found in Ref. [9(b)] exist also at small masses and can be described in the framework of the ADS effective Lagrangian. We present the simplest such asymmetric solution.

In the last section, we discuss the relevance of these new findings for the dynamics of pure supersymmetric YM theory, including the toron controversy.

## II. TENACIOUS WALLS AT LARGE MASSES

Let us consider first flavor-symmetric walls with the ansatz (7) for the moduli  $\mathcal{M}_{ij}$ . The superpotential (9) acquires the form

$$W = \frac{2}{3} \Phi^3 [\ln(\Phi^{3(N_c - N_f)} X^{2N_f}) - (N_c - N_f)] - \frac{m N_f}{2} X^2. \quad (11)$$

The corresponding scalar field potential is

$$U(\phi, \chi) = \left| \frac{\partial W}{\partial \phi} \right|^2 + \left| \frac{\partial W}{\partial \chi} \right|^2 = 4|\phi|^2 \ln(\phi^{3(N_c - N_f)} \chi^{2N_f})|^2 + N_f^2 \left| \frac{4\phi^3}{3\chi} - m\chi \right|^2. \quad (12)$$

We are set to study the wall which interpolates between the vacua:

$$\phi^3: R_*^3 \rightarrow R_*^3 e^{2\pi i N_f / N_c}; \quad \chi^2: \rho_*^2 \rightarrow \rho_*^2 e^{2\pi i N_f / N_c}, \quad (13)$$

with

$$R_*^3 = \left( \frac{3m}{4} \right)^{N_f / N_c}, \quad \rho_*^2 = \left( \frac{3m}{4} \right)^{N_f / N_c - 1}. \quad (14)$$

The BPS equations for the wall have the form

$$\partial_z \phi = e^{i\delta} 2\bar{\phi}^2 \ln(\bar{\phi}^{3(N_c - N_f)} \bar{\chi}^{2N_f}), \quad \partial_z \chi = e^{i\delta} N_f \left[ \frac{4\bar{\phi}^3}{3\bar{\chi}} - m\bar{\chi} \right] \quad (15)$$

with  $\delta = \pi N_f / N_c - \pi/2$ . There is an integral of motion

$$\text{Im}[W e^{-i\delta}] = \text{Re}[W e^{-i\pi N_f / N_c}] = \text{const.} \quad (16)$$

The energy of the wall is

$$\epsilon = 2|W_\infty - W_{-\infty}| = \frac{4N_c R_*^3}{3} |e^{2\pi i N_f / N_c} - 1| = \frac{8N_c R_*^3}{3} \sin \frac{\pi N_f}{N_c}. \quad (17)$$

Note that the phase of the fields  $\phi$  and  $\chi$  must change along the wall in such a way that the phase of the argument of the logarithm in Eq. (12) remains zero at  $z = -\infty$  as it is at  $z = \infty$ :

$$(N_c - N_f) \Delta \arg[\phi^3] + N_f \Delta \arg[\chi^2] = 0. \quad (18)$$

For large masses the matter field  $\chi$  tends to get frozen in such a way that the potentially large second term in Eq. (12) vanishes:

$$\chi^2 = 4\phi^3 / (3m). \quad (19)$$

The effective potential for the light field  $\phi$  acquires the VY form

$$U^{VY}(\phi) = 4N_c^2 |\phi|^2 \ln(\phi^3 / R_*^3)|^2. \quad (20)$$

The potential (20), as it is written, has only one minimum at  $\phi^3 = R_*^3$  and not  $N_c$  minima as we expect it to have. The resolution of this apparent paradox is well known [2]: one should take different branches of the logarithm at different values of  $\phi^3$ . The branches are glued together<sup>1</sup> at

$$\phi^3 = R_*^3 \exp\{i\pi(1 + 2k)/N_c\}, \quad k = 0, \dots, N_c - 1.$$

<sup>1</sup>Glued potentials are not specific for supersymmetric theories and also appear in the Schwinger model [10].

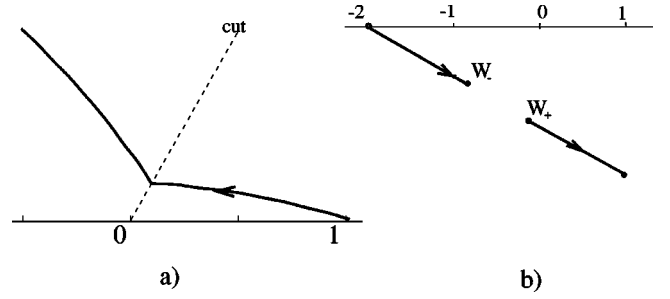


FIG. 1. Argand plots for (a)  $\phi^3/R_*^3$  and (b)  $W(\phi^3)/R_*^3$  for the BPS domain walls in the effective VY theory;  $N_c = 3$ .  $W_\pm$  are the values of the superpotential on the opposite sides of the cut.

Each branch has its own vacuum with  $\langle \phi^3 \rangle_k = R_*^3 e^{2\pi i k / N_c}$ . Actually, one can see how the branches and the branch cuts appear in the framework of the TVY model. The point is that the condition (19) cannot be satisfied *everywhere*: it would contradict the requirement (18). The only way for the solution to satisfy both contradicting requirements is the following: the relation (19) holds almost everywhere in the wall but for the narrow central region, where the field  $\chi$  changes rapidly such that

$$\Delta_{\text{core}} \arg[\chi^2] = -2\pi. \quad (21)$$

As a result of such a change, the argument of the logarithm in the effective theory (20) is multiplied by  $e^{-2\pi i N_f / N_c}$  and this exactly corresponds to crossing the branch cut and going over to another branch of the glued potential.

This scenario works, indeed, in many cases. It is clearly seen from the numerical solutions of Refs. [9,8]. Take Figs. 1 and 2 of Ref. [9]. One can see that the variable  $R = |\phi|$  just follows the solution of the effective VY theory. The variable  $\rho = |\chi|$  is frozen according to Eq. (19) everywhere but in the central region, where it undergoes a rapid change. The same concerns the phase  $\beta$  of the variable  $\phi$  vs the phase  $\alpha$  of the variable  $\chi$ .

To acquire further understanding, we plot in Fig. 1 the Argand plots for  $\phi(z)$  and for the superpotential  $W[\phi(z)]$  in VY theory.  $W$  changes along a straight line due to the property (16). We see that the superpotential is discontinuous on the cut.

The value of  $\phi^3$  on the cut is given by

$$(\phi^3)_0 = R_*^3 \eta e^{i\pi N_f / N_c}, \quad (22)$$

where  $\eta$  satisfies the condition

$$\eta(\ln \eta - 1) = -\cos \frac{\pi N_f}{N_c}, \quad (23)$$

which is a corollary of Eq. (16).

Now, in TVY theory with large but finite  $m$  there is no discontinuity, but a narrow transitional region. Within the core one can assume that  $\phi^3$  is given by Eq. (22) and is constant. It is convenient to introduce

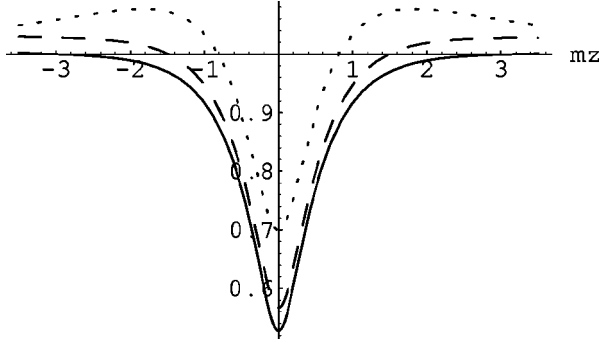


FIG. 2. Dynamics of the the field  $\rho_\zeta(z)$  inside the core:  $N_c = 3$ ,  $m = 50$  (dotted),  $N_c = 3$ ,  $m = 250$  (dashed),  $m \rightarrow \infty$  (solid).

$$\zeta = \sqrt{\frac{3m}{4\phi^3}} \chi. \quad (24)$$

The equation describing the dynamics of  $\zeta$  in the core has the universal form

$$\partial_z \zeta = -im \left( \frac{1}{\bar{\zeta}} - \bar{\zeta} \right). \quad (25)$$

The solution to Eq. (25) with boundary conditions  $\zeta(\pm\infty) = 1$  can easily be found with MATHEMATICA. [See Fig. 2, where it is plotted together with the right side of Eq. (24) obtained from the numerical solutions of the BPS equations in TVY theory for large but finite  $m$ .] The phase  $\gamma$  ( $\zeta \equiv \rho_\zeta e^{i\gamma}$ ) is changed by  $-\pi$ . There is an integral of motion

$$\ln \rho_\zeta - \frac{\rho_\zeta^2}{2} \cos(2\gamma) = \text{const} = -\frac{1}{2}. \quad (26)$$

In the center of the wall,  $\gamma = -\pi/2$  and  $\rho_\zeta \approx 0.52$ . We see from Fig. 2 that, for large masses, the dependence  $|\chi(z)|$  inside the core is indeed determined by Eq. (25). Also,  $\chi(z)$  satisfies the condition (19) in the coat.

Let us determine the fraction of the energy of the wall stored in its coat. It is given by the expression

$$f_{\text{coat}} = \frac{|W_\infty - W_+| + |W_- - W_{-\infty}|}{|W_\infty - W_{-\infty}|}, \quad (27)$$

where  $W_\pm$  are the values of the superpotential at the opposite sides of the core. A simple calculation using, again, the condition (16) gives

$$f_{\text{coat}} = \frac{|\sin(\pi N_f/N_c) - \pi \eta N_f/N_c|}{\sin(\pi N_f/N_c)}. \quad (28)$$

Let us look at Eq. (23) determining the parameter  $\eta$ . At  $N_f/N_c < 1/2$  it has two real roots. One of them ( $\eta_1$ ) is smaller than 1 and the corresponding fraction (28) is also less than 1. Another root ( $\eta_2$ ) lies within the range  $1 < \eta_2 < e$ . For  $N_c < 5$ , the corresponding fraction is greater than 1, which obviously means that this solution is not acceptable. But even for  $N_c > 5$  when  $f_{\text{coat}}$  as determined by Eq. (28) is less than 1, the root  $\eta_2$  does not correspond to any wall

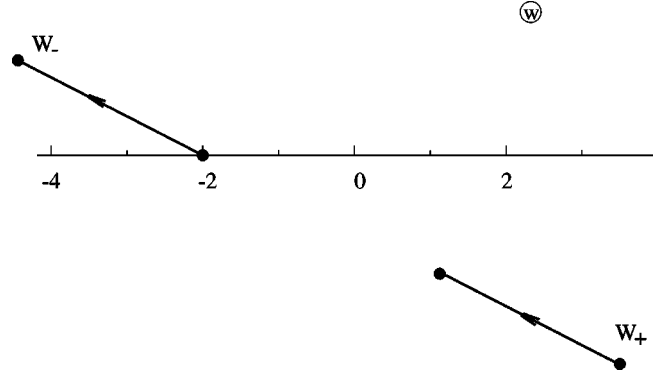


FIG. 3. The coat for a nonexistent wall ( $N_c = 3$ ).

solution in TVY theory. Again, the picture can be clarified by drawing the Argand plot for the corresponding BPS solutions in VY theory (see Fig. 3). The differences  $W_\infty - W_+$ ,  $W_- - W_{-\infty}$  have now the opposite sign as compared to the previous case, and one can no longer pass from  $W_-$  to  $W_+$  moving in the positive  $z$  direction.

If  $N_f/N_c = 1/2$ ,  $\eta_1 = 0$ . This corresponds to the wall passing through the chirally symmetric vacuum in the middle. This is, indeed, the only way for different chirally asymmetric vacua to be connected in the theory with  $N_c = 2$ ,  $N_f = 1$  for large masses [5]. For  $N_f/N_c > 1/2$ , the real root  $\eta_1$  disappears and there is no solution whatsoever.

If  $N_f = 1$  and  $N_c$  is large,  $\eta \approx 1 - \pi/N_c$  and

$$f_{\text{coat}}^{1 \text{ flavor, large } N_c} \approx \frac{\pi}{N_c} \ll 1, \quad (29)$$

i.e., almost all energy is stored in the core. This agrees with the analysis in Ref. [11], where the mechanism of regularizing the branch cut singularity by “integrating in” an extra heavy field was first suggested. (The authors of [11] did not analyze TVY theory, however, and restricted themselves to discussion of toy models.) If not only  $N_c$  but also  $N_f$  is large, the arguments of [11] do not apply and  $f_{\text{coat}}$  is not necessarily small. The argument based on analysis of the expression (28) gives an explanation why flavor-symmetric walls do not exist when  $N_f/N_c > 1/2$  and  $m$  is large, which is complementary to that in Refs. [9].

It was noticed that, if the requirement (7) is relaxed, tenacious domain walls exist even in the range  $N_f/N_c > 1/2$ . Consider the simplest case  $N_c = 3$ ,  $N_f = 2$ . Assume  $\mu_{ij} = \text{diag}(\chi_1^2, \chi_2^2)$ ,  $\chi_1 \neq \chi_2$ . A tenacious BPS solution with

$$\Delta \arg[\phi^3] = \Delta \arg[\chi_1^2] = \frac{2\pi}{3}, \quad \Delta \arg[\chi_2^2] = -\frac{4\pi}{3} \quad (30)$$

exists. In the large mass limit  $\chi_1^2$  stays frozen according to Eq. (19) *everywhere*, while the field  $\chi_2^2$  undergoes a rapid change in the core, which is described by the universal equation (25). Using the terminology of Ref. [9(b)], the field  $\chi_2$  has a nontrivial *winding*, while  $\chi_1$  has not. As  $\Delta \arg[\phi^3]$  is the same in this case as in the theory with  $N_c = 3$ ,  $N_f = 1$ , the Argand plots for  $\phi^3$  and  $W(\phi^3)$  in the effective VY theory



are also the same and are given in Fig. 1. The wall exists for arbitrary large masses. The fraction of the energy stored in the coat is given by the same formula (28), with  $N_f$  substituted by

$$k = \sum_i^{N_f} \omega_i, \quad (31)$$

where  $\omega_i$  are the windings of the matter fields. They acquire values 0 or 1. A wall with  $k/N_c < 1/2$  is tenacious.

### III. TENACIOUS WALLS IN THE ADS LIMIT

The main characteristic feature of the tenacious solutions is that they persist for arbitrary large masses. But, of course, they also exist in the small mass limit, where the system is described by the ADS effective Lagrangian with the superpotential (3). The latter is obtained from Eq. (9) by freezing the heavy field  $\Phi$  so that the argument of the logarithm is equal to 1.

Note that fractional powers in Eq. (3) do not give rise to a new kind of glued potential. The point is that the domain wall solutions always stay on the same sheet of the function (3) and the problem of discontinuities associated with branch cuts does not arise. Suppose, e.g., that  $N_f = 1$ . Then

$$W = -\frac{2(N_c - 1)}{3(X^2)^{1/(N_c - 1)}} - \frac{m}{2}X^2. \quad (32)$$

Let us choose the sheet where  $W$  is real for real positive  $X^2$ . Then a perfectly smooth BPS domain wall interpolating between  $\chi^2 = \rho_*^2$  and  $\chi^2 = \rho_*^2 e^{2\pi i/N_c}$  exists such that

$$\Delta \arg[\chi^2] = \frac{2\pi}{N_c} - 2\pi, \quad \Delta \arg[(\chi^2)^{-1/(N_c - 1)}] = \frac{2\pi}{N_c}. \quad (33)$$

One can recall here that, although the theories with  $N_f = N_c - 1$  are somewhat nicer because all gauge fields become heavy and we are in the Higgs weak coupling regime, the mass of the lowest excitations is of order  $m \ll \Lambda_{\text{SQCD}}$  for any  $N_f$ , and the ADS effective Lagrangian always has a Wilsonian nature.

The ADS Lagrangian does not describe, however, the chirally symmetric sector, where  $\Phi^3 \equiv 0$  and the effective superpotential is just

$$W_{\text{eff}}^{\text{chir. inv.}}(\mathcal{M}) = -\frac{m}{2} \text{Tr } \mathcal{M}. \quad (34)$$

The walls that penetrate into this sector also have, for small masses, a multilayer matryoshka structure similar to the structure of tenacious walls in the large mass limit, discussed above, but the role of the heavy and light fields is now reversed.

This especially concerns the “lower BPS branch” for the flavor-symmetric nontenacious walls. As was shown in Sec. VII of Ref. [6], the wall in this case consists of *five* layers: two wide (with characteristic width  $\propto 1/m$ ) outer layers

whose dynamics is described by the ADS Lagrangian, a wide central region, where  $\phi^3$  is close to zero and there is only the quadratic term  $\propto |\bar{\chi}\chi|$  in the effective potential, and two narrow (with the characteristic width  $\propto \Lambda_{\text{SQCD}}$ ) transitional regions, where the field  $\phi^3$  changes rapidly, while the matter field  $\chi$  stays effectively frozen.

Also “real” walls interpolating between the chirally symmetric and chirally asymmetric vacua consist of two layers: a narrow one, where the field  $\phi^3$  changes rapidly, and a wide layer, where  $\phi^3 \approx 0$  and only the matter field changes.

The ADS Lagrangian also admits flavor-asymmetric wall solutions. Consider the case  $N_c = 3$ ,  $N_f = 2$ ,  $\mathcal{M}_{ij} = \text{diag}(X_1^2, X_2^2)$ . The superpotential is

$$W = -\frac{2}{3X_1^2 X_2^2} - \frac{m}{2}(X_1^2 + X_2^2). \quad (35)$$

Consider the wall where the phases of the fields  $\chi_1^2, \chi_2^2$  change in opposite directions:

$$\chi_1^2: \rho_*^2 \rightarrow \rho_*^2 e^{2\pi i/3}; \quad \chi_2^2: \rho_*^2 \rightarrow \rho_*^2 e^{-4\pi i/3}. \quad (36)$$

The corresponding BPS equations

$$\begin{aligned} \partial_z \chi_1 &= e^{-i\pi/6} \left[ \frac{4}{3\bar{\chi}_1^3 \bar{\chi}_2^2} - m\bar{\chi}_1 \right], \\ \partial_z \chi_2 &= e^{-i\pi/6} \left[ \frac{4}{3\bar{\chi}_2^3 \bar{\chi}_1^2} - m\bar{\chi}_2 \right] \end{aligned} \quad (37)$$

can be solved. The profiles of  $\rho_1 = |\chi_1|$  and  $\rho_2 = |\chi_2|$  are presented in Fig. 4.

The presence of flavor-asymmetric domain walls is a rather remarkable and nontrivial fact. Note that their presence in the Wilsonian ADS Lagrangian *assures* their existence in SQCD. Flavor-symmetric and flavor-asymmetric domain walls exist in the ADS limit for any  $N_f$  and any combination of windings. Lifting them up to TVY theory, we observe that, for low masses, the argument of the logarithm in Eq. (9) is close to 1 and its phase to zero. Therefore, in contrast to the authors of Ref. [9(b)], we would not call “unphysical” the solutions with  $k > 1$  on the ground that the phase of the logarithm changes by  $\Delta = 2\pi k > 2\pi$  in the central region of the wall in the large mass limit. They are certainly physical at small masses when  $\Delta = 0$ . Then  $\Delta$  increases with increasing mass up to  $2\pi k$  for  $m \rightarrow \infty$ , but we do not see in what respect the solution with  $\Delta = 359^\circ$  at somewhat smaller mass is better than the solution at somewhat larger mass with  $\Delta = 361^\circ$ . Both solutions are smooth and do not have problems with discontinuities associated with branch cuts. (One should be very careful indeed when a solution runs into such a discontinuity.) Unlike the case of flavor-symmetric walls at  $N_f = N_c - 1$  analyzed in Refs. [5,6], no phase transition in mass occurs here.

In the case  $N_c = 3$ ,  $N_f = 2$ ,  $k = 1$ , we have three walls: two flavor-asymmetric walls and the flavor-symmetric one. For arbitrary  $k$  and  $N_c$  ( $N_f = N_c - 1$ ), the number of different walls can be determined by adapting the arguments of Ref.

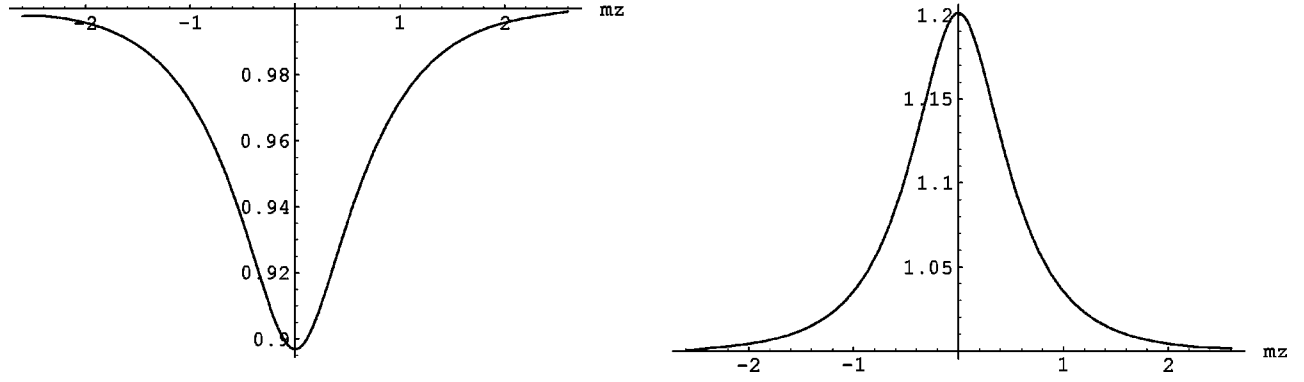


FIG. 4. Asymmetric wall in the BPS limit ( $N_c=3$ ,  $N_f=2$ ,  $k=1$ ): the plots of  $\rho_1(z)/\rho_*$  (left) and  $\rho_2(z)/\rho_*$  (right).

[12], where the number of solitons in the supersymmetric  $CP^{N-1}$  model was calculated.<sup>2</sup> [The relevant effective superpotential for  $CP^{N-1}$  coincides [13] with the ADS Lagrangian in the framework of the diagonal ansatz (7).] Let us write the superpotential as

$$W = -\frac{m}{2} \sum_{i=1}^{N_c} X_i^2, \quad (38)$$

where

$$X_{N_c}^2 \stackrel{\text{def}}{=} -\frac{4}{N_c-1} \cdot \frac{1}{3m \prod_{i=1}^{N_c} X_i^2}.$$

The wall interpolates between the points with the values of the superpotential  $W_*$  and  $W_* \exp\{2\pi i k/N_c\}$ . Now, for each  $\chi_i^2$  we have

$$\Delta \arg[\chi_i^2] = 2\pi \left( \frac{k}{N_c} - \omega_i \right) \quad (39)$$

with  $\omega_i=0$  or  $\omega_i=1$ . Bearing in mind that  $\Delta \arg[\prod_{i=1}^{N_c} \chi_i^2] = 0$ , there are  $k$  fields with  $\omega_i=1$  and  $N-k$  fields with  $\omega_i=0$ . Altogether, there are  $C_k^{N_c}$  possibilities.

This was done in the framework of the diagonal ansatz (7). Allowing for flavor rotations and assuming the mass matrix to be diagonal, we obtain not a discrete set of walls, but continuous families of solution (the wall moduli space). For a generic mass matrix and if transverse directions are compactified, the degeneracy of this moduli space is lifted, however, and we are left with  $C_k^{N_c}$  isolated solutions [14]. Remarkably, this coincides with the estimate for the number of walls in SYM theory compactified on  $T^2$ , obtained in Ref. [15] using  $D$ -brane arguments.

We should emphasize again, however, that this counting does not work for the TVY model. In the theory with  $N_f = N_c - 1$ , there are more than  $C_k^{N_c}$  walls in the limit of small

masses and fewer than  $C_k^{N_c}$  tenacious walls surviving for large masses. [The number of the latter is equal to  $C_k^{N_c-1}$  if  $k < N_c/2$  and to zero otherwise — see Eq. (31).]

#### IV. DISCUSSION

For us, the main interest of the study of domain walls in supersymmetric QCD is a hope of shedding light on the long-standing “toron controversy,” associated with the vacuum structure of pure SYM theory. Two different interpretations of the basic relation (1) are possible.

(1) The chiral symmetry  $U_A(1)$  of the free SYM action is explicitly broken down to  $Z_{2N_c}$  due to anomaly. Further,  $Z_{2N_c}$  is spontaneously broken down to  $Z_2$ . The condensate (1) plays the role of order parameter associated with this breaking.

(2)  $U_A(1)$  is explicitly broken down to  $Z_2$  and the different vacua (1) lie in different sectors of the Hilbert space. The integer  $k$  plays the same role in this case as the parameter  $\theta$  or, better said,  $\theta$  changes within the range  $(0, 2\pi N_c)$  and not within the range  $(0, 2\pi)$  as it does in standard QCD. This implies the relevance of “torons”—configurations with fractional topological charges which exist in a finite four-dimensional box [16] and might stay relevant also in the limit when the size of the box is sent to infinity [17]. All the arguments for and against were discussed anew recently in Ref. [4].

The first picture (complemented with the assumption that the chirally symmetric vacuum is an artifact of the VY approach and is not really there) is standard.<sup>3</sup> This necessarily implies the presence of physical domain walls interpolating between the vacua with different phases of  $\langle \text{Tr} \{\lambda^2\} \rangle$ . If the second picture is correct, there are no such domain walls.

At the moment, one cannot say with certainty whether these domain walls exist or not in pure SYM theory. There are  $D$ -brane arguments in favor of their existence [19], but it is important to try to resolve this field theory issue within the field theory framework. In early works [5,6], it was shown that a certain type of wall present in supersymmetric QCD

<sup>2</sup>I am indebted to T. ter Veldhuis, M. Shifman, and A. Vainshtein for the discussion of this point.

<sup>3</sup>One of the arguments is the absence of the chirally symmetric state and irrelevance of torons in  $\mathcal{N}=2$  SYM theory [18].

disappears in the large mass limit. The results of Refs. [9,8] and of the present work show that there are tenacious walls, which persist in the large mass limit. Note, however, that the core of such a wall becomes very thin in this limit and the energy density in the core becomes very large. We find this situation rather strange. The assumption that these walls with narrow dense core are relevant for physics in pure SYM theory contradicts the common wisdom that heavy fields should decouple in the limit  $m \rightarrow \infty$  and have no effect on the dynamics of the low energy sector.

We *think* that this general argument should work also in this case, but, obviously, further study of this question is required.

The last comment concerns the dynamics of SYM theory at finite temperature. Even though supersymmetry is broken by temperature,<sup>4</sup> one can use temperature as a theoretical tool to distinguish between two different scenarios of the chiral symmetry breaking in SYM theory mentioned above. It is not easy, however.

In the first scenario (with walls), the spontaneously broken discrete chiral symmetry is restored at some critical tem-

perature  $T_c$ , by the same token as the spontaneously broken continuous chiral symmetry  $SU_L(N_f) \times SU_R(N_f)$  is restored in the standard massless QCD. If one assumed that complex domain walls decouple in the large mass limit *and* the presence of the chirally symmetric vacuum were disregarded, one would conclude that the chiral condensate retained a nonzero value for any temperature (as it does in QCD with one light flavor) and there would be no phase transition. However, in the TVY model different chirally asymmetric vacua *can* communicate with each other with the chirally symmetric state as an intermediary. In practice, this means that the chiral symmetry is duly restored at some  $T_c$  in the same way as it is in the standard scenario.

To conclude, decoupling of complex walls in the large mass limit implies *either* appearance of a new superselection rule for the parameter  $k$  and the relevance of fractional topological charges in pure SYM theory *or* the presence of the chirally symmetric vacuum state. The TVY/VY approach favors the second possibility.

### ACKNOWLEDGMENTS

I am indebted to D. Binosi, M. Shifman, A. Vainshtein, and T. ter Veldhuis for illuminating discussions.

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<sup>4</sup>Contrary to what many people think, this breaking is not explicit, but spontaneous [20].

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